The SINE/COSINE Integer Harmonic Ratios of ANGLES Using a Cartesian Coordinate System

by Benjamin Robert Tubb (7 March 2008)
Email: brtubb@pdmusic.org

		90		
		1,0		
180	-1,0	0,0	1,0	0
		-1,0		
		270		

135				45
	-1,1		1,1	
		0,0		
	-1,-1		-1,1	
225				315

From the above two things should be noticable, (1) the coordinates are "reflected" by their abssolute values around a circle, such that the values around the center (0,0) each describe angle displacements of 45 degree multiples; and (2) that only the values of one quadrant can be used to give various eightfold symmetry around the 8 semi-quadrants.of the "basic" four!

Since 360 degrees are defined as a circular closed arc, its integer factors are especially important to know, and thus allow an alternative method of defining those arcs (angles) in terms of a coodinate "displacement" which in effect can be used to define ANY trigonometric integer ratio for SINE or COSINE. Remember that the length of the hypontenuse of a right triangle is equal to the square root of the sum of the squares of its sides, thus in x, y cordinates with a circle's center "displaced" to the coordinates of (0,0), its radius becomes equal to such a hypotenuse for any given equal values of x and y. Therefore ALL other angular values can be interpolated based in that relationship, for which those integers which are factors of 360 are those which will give integer angles with appropriate integer factors. The integer prime factors of 360 are 2^3 * 3^2 * 5, which give the following integer angle multiples 1, 2, 3, 4, 5, 6, 8, 9, 10, 15, 20, 30, 45, 60, 90, 120, and 180.

For example the 90 degree (1,0), (0,1), (0,-1), (-1,-1) displacemts around the origin (0,0) equal respectively 90, 180, 270 and 360 (=0).

Therefore the 45 degree (1,1) displacements around a 360 degree full circle are excluding their common multiples for the previous ones, are at 45, 135, 225, and 315 degrees.

So what does this all suggest? To define a circle with x,y values represenative of each 1 degree angle in a circle, a coordinate system grid must contain at least 120 x and y coordinates with respect to a given 0,0 orgin. To further define so-called minutes and seconds then larger cooridinate grids must be used of respectively 180 by 180 for minutes and 240 by 240 for seconds.

Using the tables the (x,y) displacements for line connections from any given origin would produce corresponding 45 degree semi-quadrant angle displacements. For example (3,2) = 60, (-3,2) = 120, (-3,-2) = 240, and (3,-2) = 330. Thus the SIGN attribute of each coordinate can be used to determine the quadrant to place the angle within. The other characteristic of these coordinate values is their comparative values. If the x coordinate is greater than the y coordinate then its 45 degree "semi-quadrant" is either from 1 to 44, 135-179, 226-269, or 316-359 . Then angles in each of the other semi-quadrants can be considered "reflections" (or reciprocals) of its values.

Integer Factors of 360

Angle	x	y
0	1	0
1	60	1
2	45	1
3	30	1
4	24	1
5	18	1
6	15	1
8	12	1
9	10	1
10	9	1
15	6	1
20	9	2
30	3	1
45	1	1
60	3	2
90	0	1
120	-3	2
180	-1	0

Extrapolations for 7 to 44 excluding the above

Angle	x	y
7		
11		
12		
13		
14		
16		
17		
18	5	1
19		
21		
29		
31		
32		
33		
34		
35		
36	5	2
37		
38		
39		
40	9	4
41		
42		
43		
44		

The musical implications of such a coordinate transformation of angles to ratios also suggests perhaps an octave frequency ratio (known to be 2/1) to be used as a means to determining scale tone tuning systems with their integer based ratio equivalents for each scale "degree" or angle!

The tables below could be used to determine the powers of two exponent to be applied to any given frequency to determine its scale degrees in ALL (audible) octaves, which is approximately, for human hearing, in the range of the 88 notes of a grand piano, which are from A0 (MIDI note 20 = 27.5Hz) to C8 (MIDI note 108 = 4186Hz).

Harmonics

	x	y
I0	0	0
I1	2	1
V1	3	1
I2	4	1
III2	5	1
V2	6	1
vii2	7	1
I3	8	1
II3	9	1
III3	10	1
#IV3	11	1
V3	12	1
vi3	13	1
	14	1
VII3	15	1
I4	16	1
ii4	17	1
II4	18	1
iii4	19	1
III4	20	1
IV4	21	1
	22	1
	24	1
	25	1
	26	1
	27	1
	28	1
	29	1
	30	1
	31	1
I5	32	1

Scale Degrees

	x	y	Ratio Values	12 Tone Equal Temperament Tuning Equivalents (2^(n/12)	
1	0	0			
b2	17	16	1.0625	1.0595	1
2	9	8	1.1250	1.1225	2
b3	19	16	1.1875	1.1892	3
3	5	4	1.2500	1.2599	4
4	4	3	1.3333	1.3348	5
#4/b5	11	8	1.3750	1.4142	6
5	3	2	1.5000	1.4983	7
#5/b6	8	5	1.6000	1.5874	8
6	13	8	1.6250	1.6818	9
b7	7	4	1.7500	1.7818	10
7	15	8	1.8750	1.8877	11
8	2	1			

Other choices

8	7	1.1429
8	5	1.6000
4	3	1.3333
5	3	1.6667
8	7	1.1429
12	7	1.7143
8	7	1.1429
10	7	1.4286
12	7	1.7143
10	6	1.6667